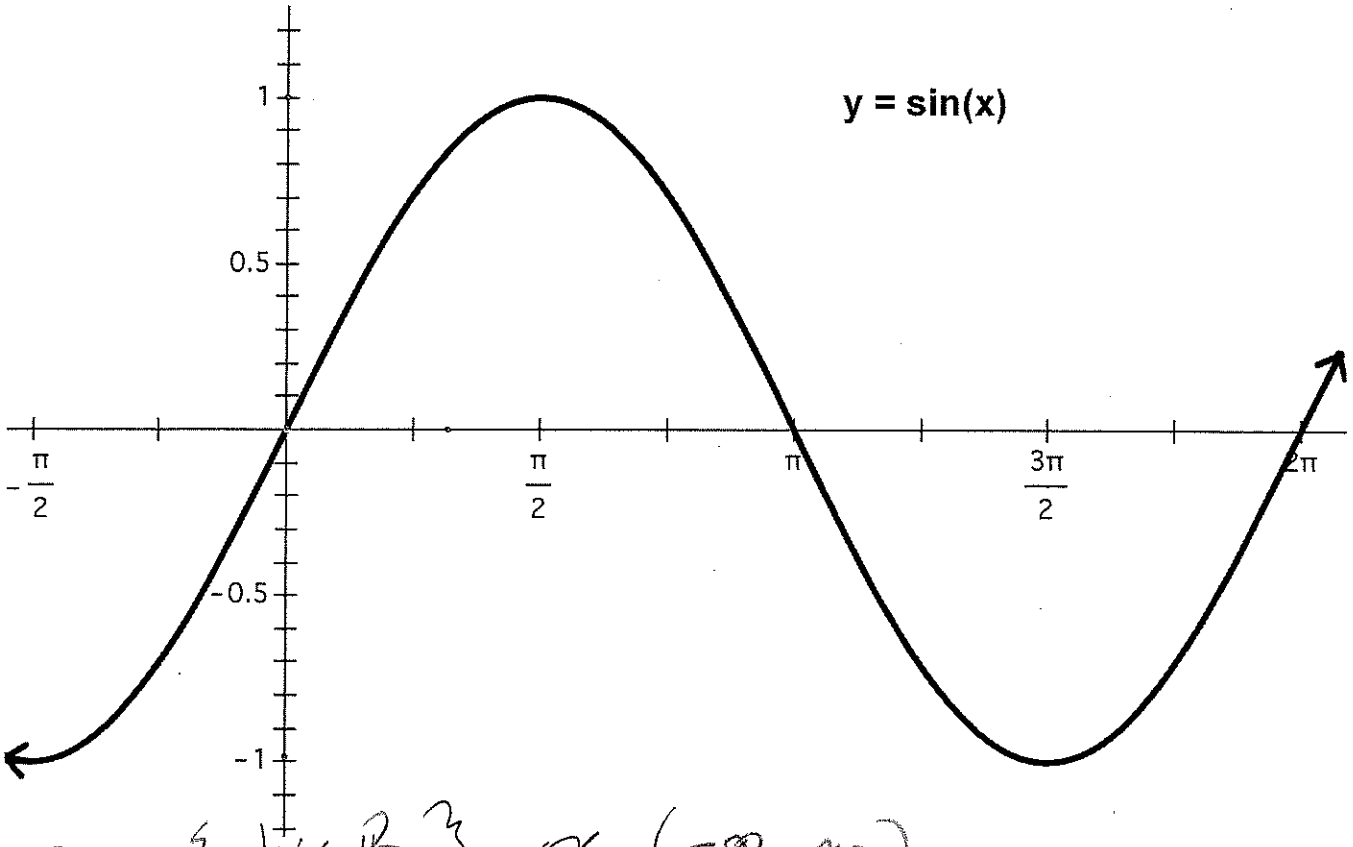
Sections 4.5 – Graphs of Sine & Cosine Functions

Sine Curve

The graph of the sine function

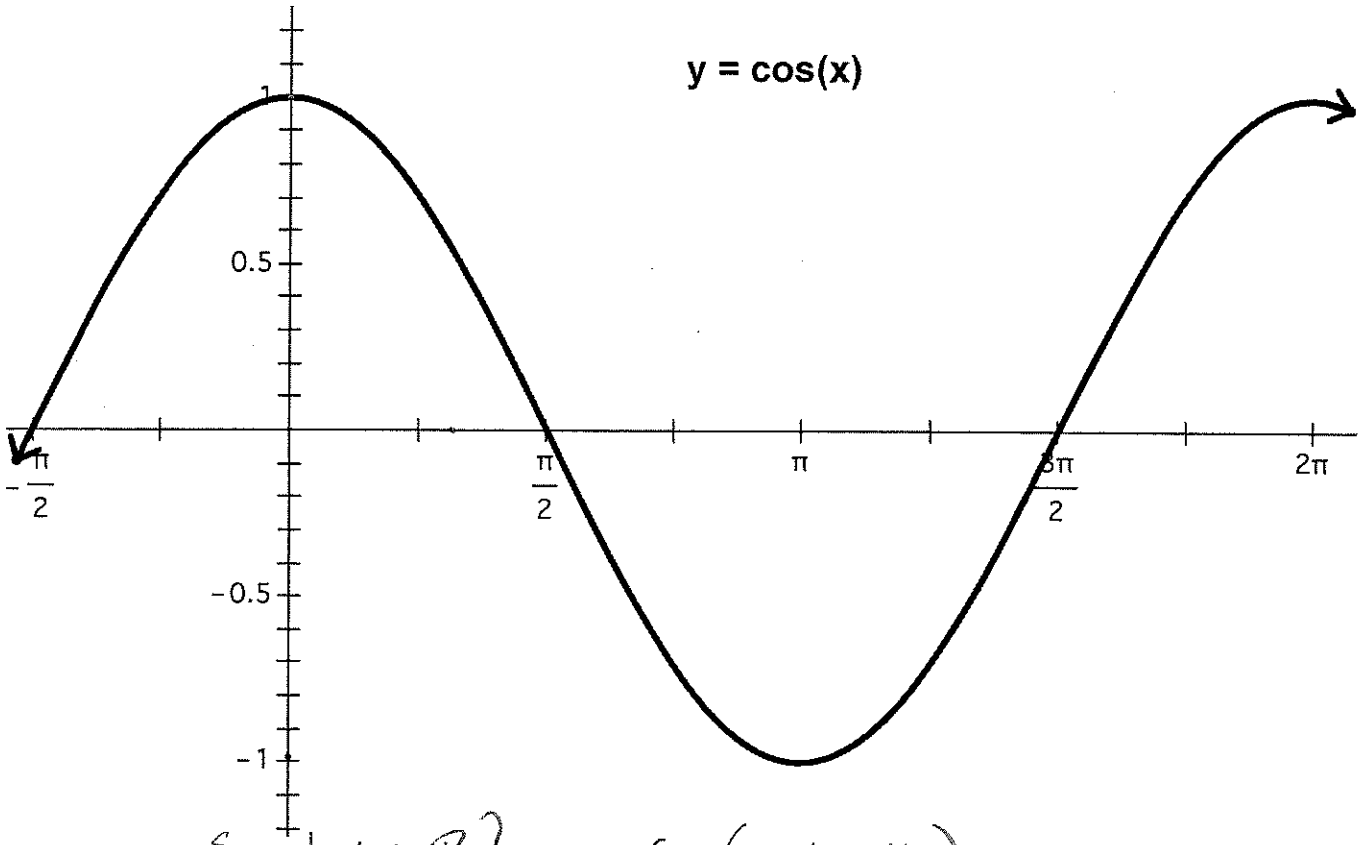


- Domain: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$
- Range: $\{y \mid -1 \leq y \leq 1\}$ or $[-1, 1]$
- Symmetric with respect to the***origin*** so this is an***ODD*** function.
- Period: 2π
- Zeros: $\ldots, -\pi, 0, \pi, 2\pi, \ldots$
- Max: 1
- Min: -1

Sections 4.5 – Graphs of Sine & Cosine Functions

Cosine Curve

The graph of the cosine function



- Domain: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$
- Range: $\{y \mid -1 \leq y \leq 1\}$ or $[-1, 1]$

- Symmetric with respect to the ***y-axis*** so an ***EVEN*** function

- Period: 2π
- Zeros: $\ldots, -\pi/2, \pi/2, 3\pi/2, \ldots$
- Max: 1
- Min: -1

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Important Vocab

Amplitude – *half the distance between the minimum and maximum value of a function over a given range.*

Period – *the distance required to complete one full cycle (one complete repetition of a pattern)*

Frequency – *the number of cycles completed in a given interval* (frequency = $\frac{1}{\text{period}}$)

Summary of Translations of Sine and Cosine

$$Y = A \sin B(x - C) + D$$

A = amplitude

B = frequency which gives us a period of $2\pi/B$

C = Horizontal shift

D = Vertical shift

The general equations,

$y = A\sin B(x-C)+D$ and $y = A\cos B(x-C)+D$

have the following characteristics:

amplitude = |A| **period =** $\frac{2\pi}{B}$

1. If C > 0 there is a horizontal shift C units to the right

   if C < 0 there is a horizontal shift C units to the left.

2. If D > 0 the shift is D units upward

   if D < 0 the shift is D units downward.

3. If A < 0 ➔ reflection across x-axis.

4. If B < 0 ➔ reflection across y-axis.

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Examples:

Describe the change (transformations) between the following graphs and their respective parent functions- consider amplitude, period and shifts:

a) $y=-4\cos(5x)$

amplitude: 4

period: $2\pi/5$

Reflection? x-axis

Shifts: none

b) $y=\frac{1}{2}\sin(6x)$

amplitude: $\frac{1}{2}$

period: $\pi/3$

Reflection? none

Shifts: none

c) $f(x)=-3\sin(x+\pi)$

amplitude: 3

period: 2π

Reflection? x-axis

Shifts: Left by π

d) $f(x)=\cos\frac{1}{4}x+8$

amplitude: 1

period: $2\pi/1/4 = 8\pi$

Reflection? none

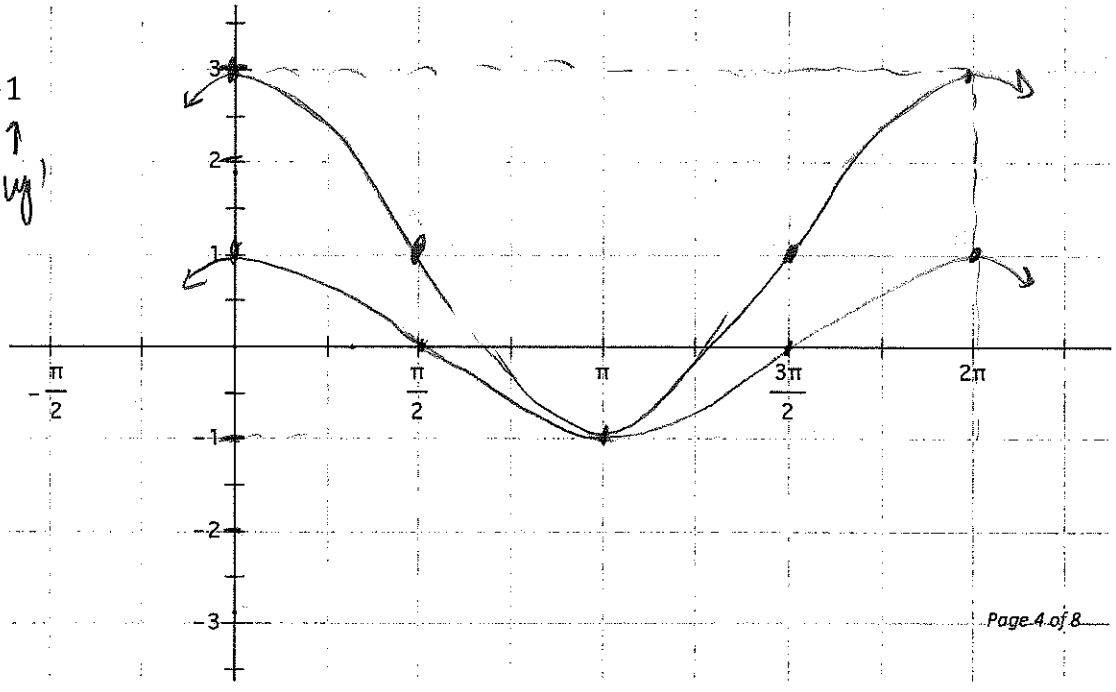
Shifts: up 8

Graph the following functions:

a) $y=\cos x$

b) $y=2\cos x+1$

↑ amp ↑ up 1

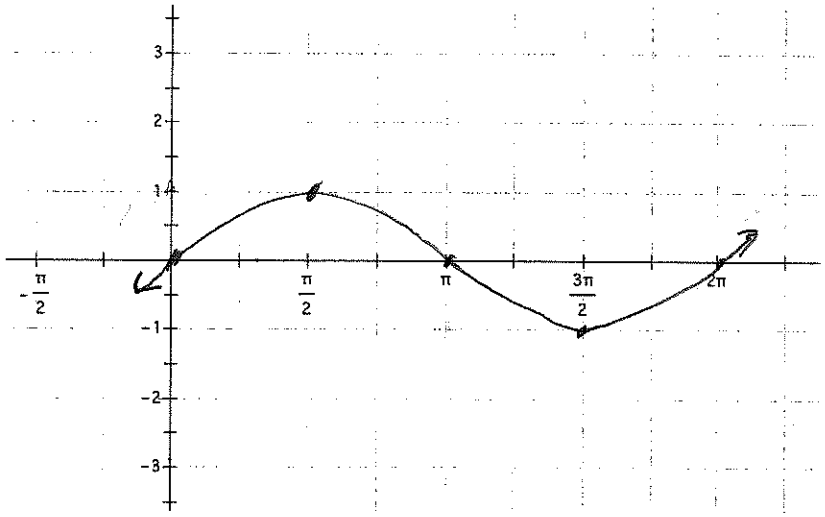


More Examples!

For the following graphs, graph on the same coordinate axes.
You should graph at least between 0 and 2π, unless you want to graph more...

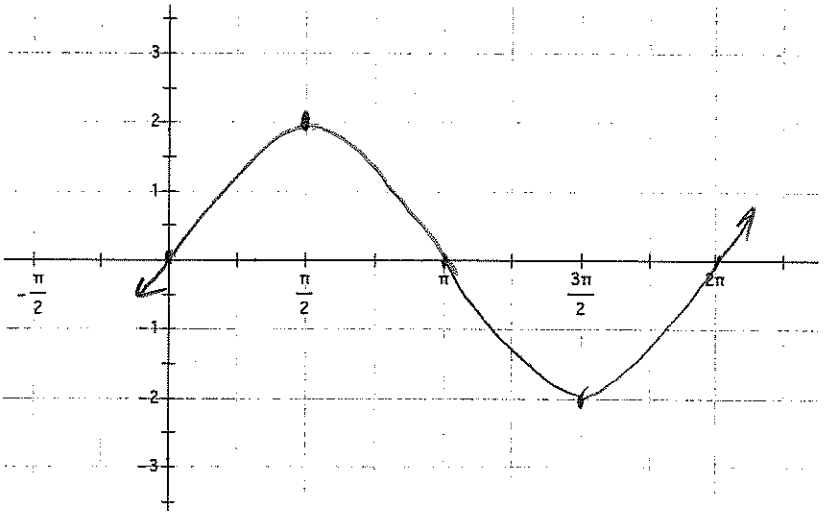
1.

a) $y = \sin x$



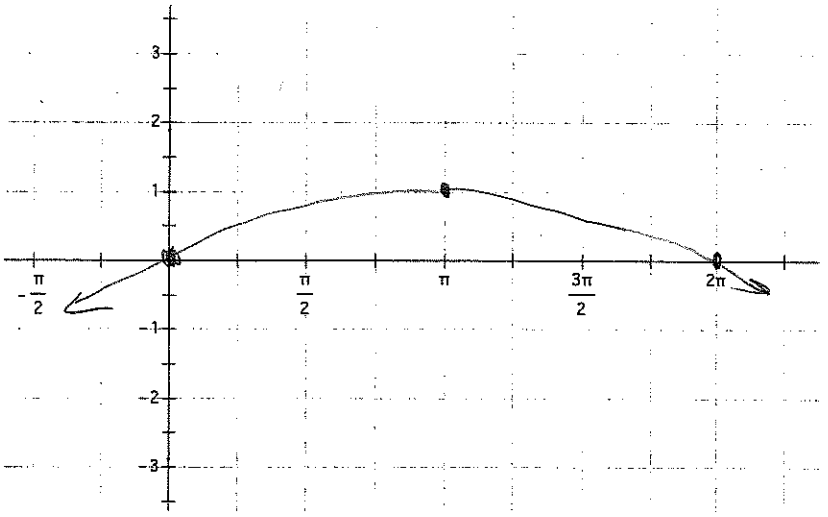
b) $y = 2\sin x$

amp! = 2



c) $y = \sin\left(\frac{1}{2}x\right)$

period = $\frac{2\pi}{1/2} = 4\pi$



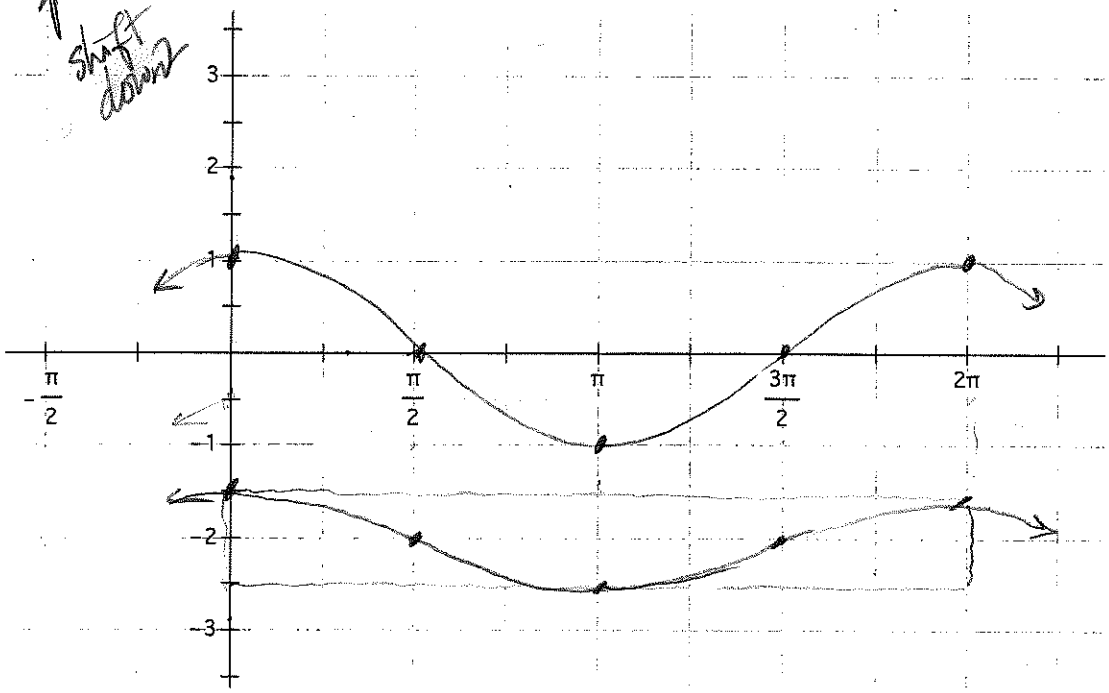
Sections 4.5 – Graphs of Sine & Cosine Functions

2.

a) $y = \cos x$

b) $y = \frac{1}{2}\cos x - 2$

amp $= \frac{1}{2}$ ↑ shift down

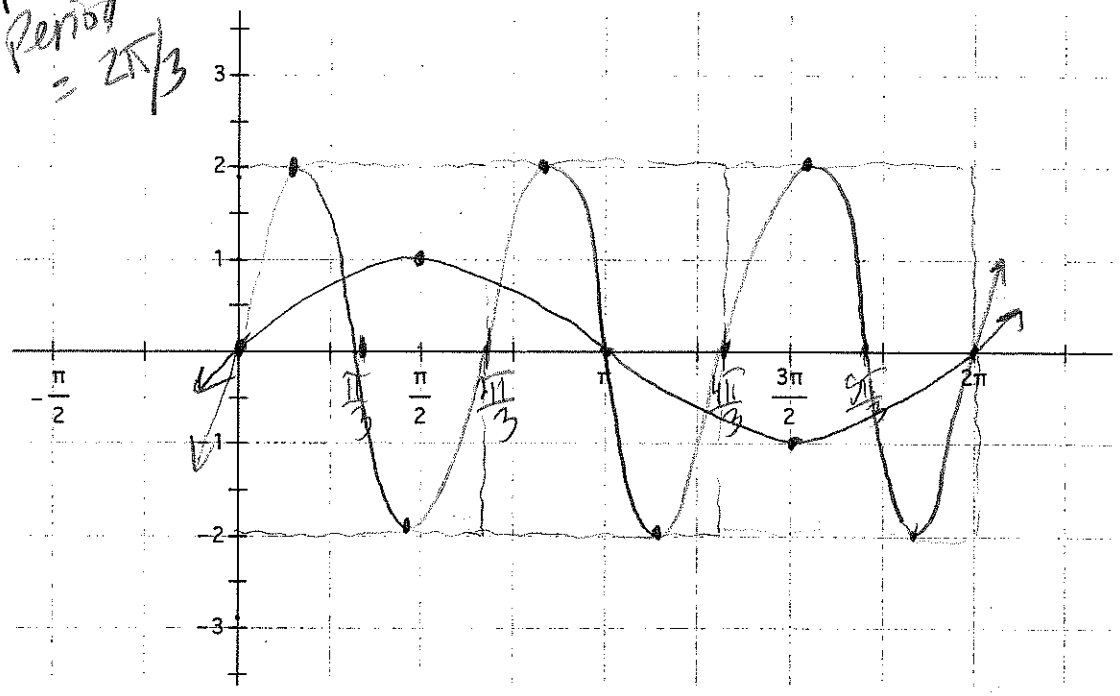


3.

a) $y = \sin x$

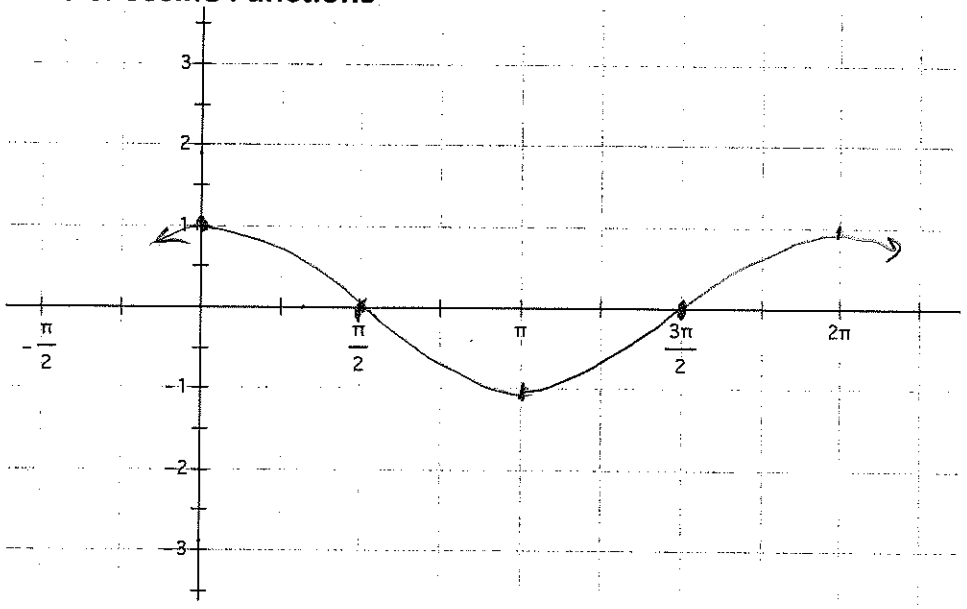
b) $y = 2\sin 3x$

amp $= 2$ ↑ Period $= 2\pi/3$



4.

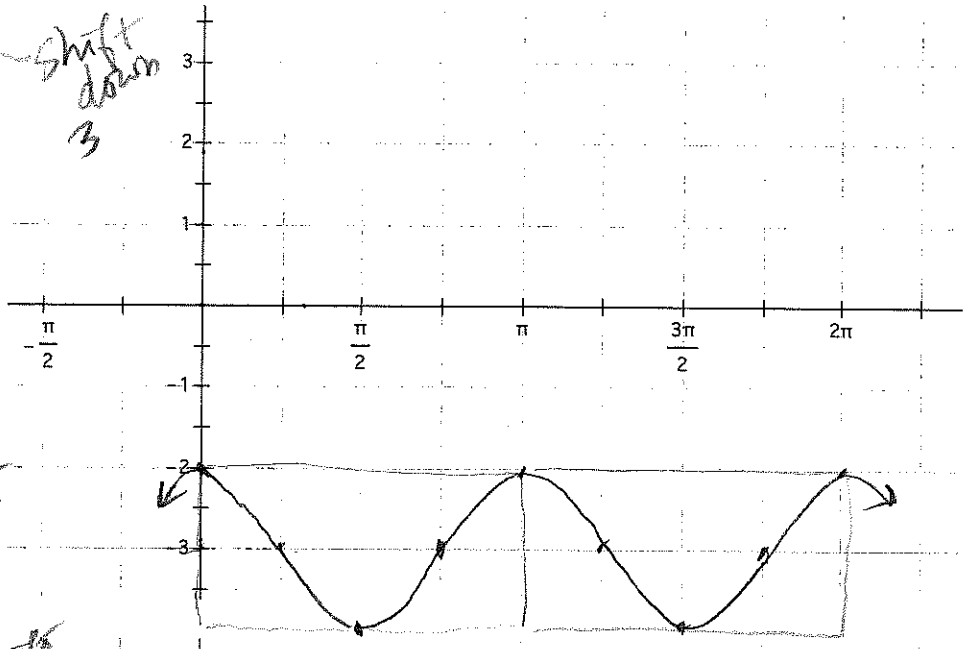
a) $y = \cos x$



b) $y = \cos 2x - 3$

$p = \frac{2\pi}{2} = \pi$

shift down 3



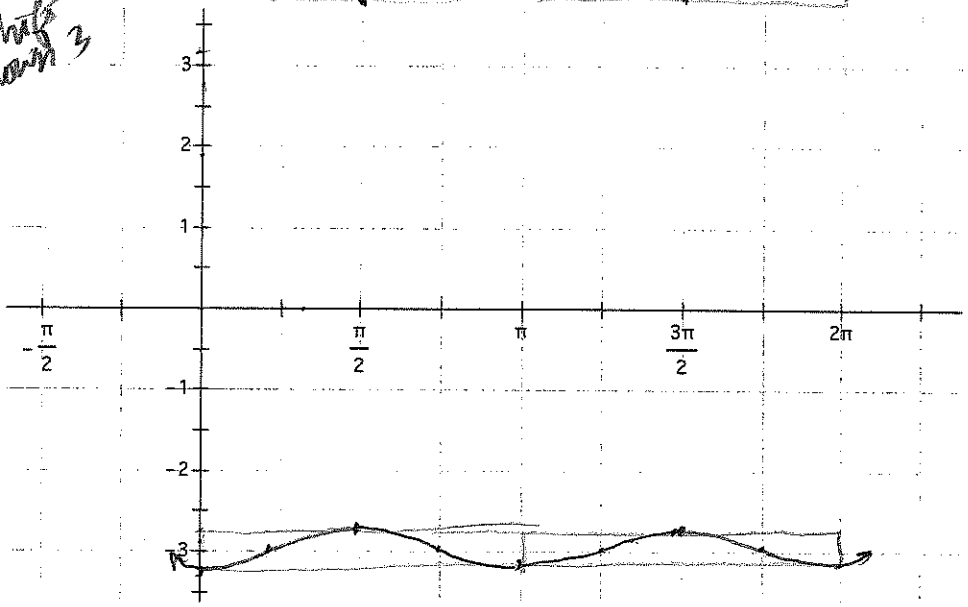
c) $y = -\frac{1}{4}\cos 2x - 3$

$p = \frac{2\pi}{2} = \pi$

shift down 3

reflect over x-axis

amp $= \frac{1}{4}$



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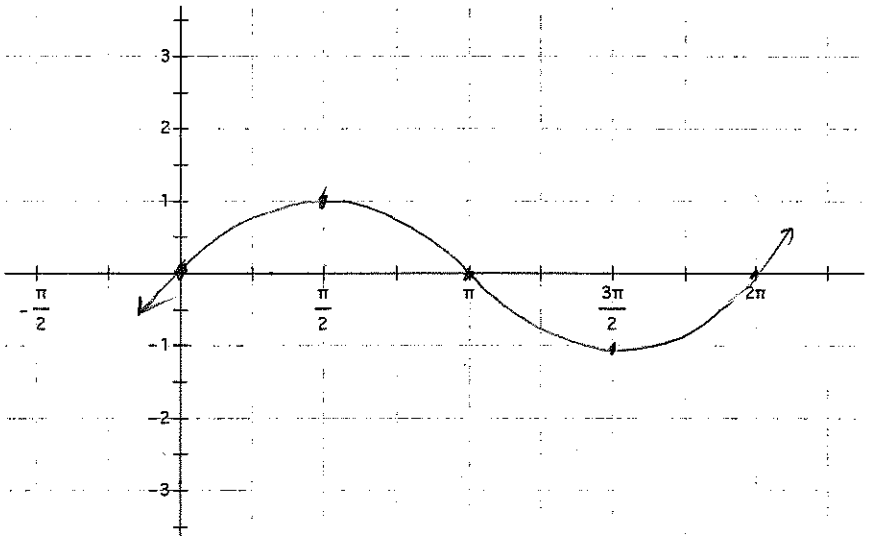
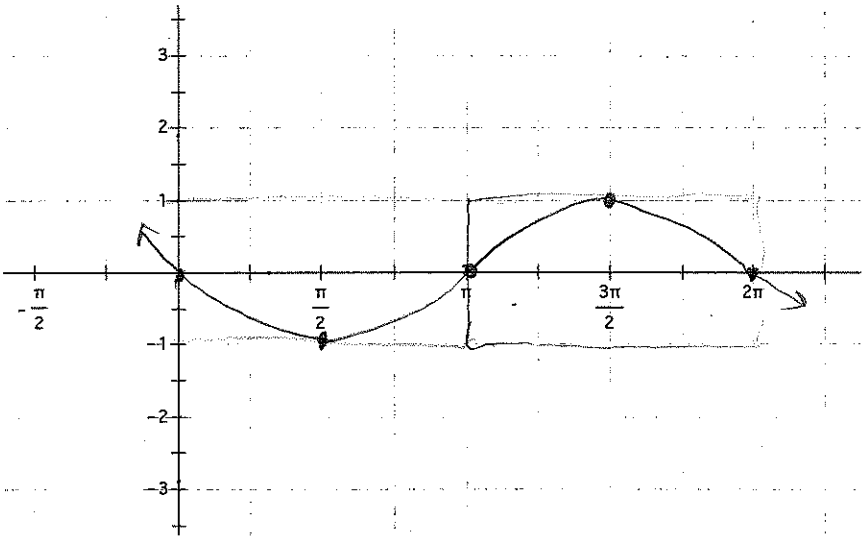
5.

a) $y = \sin x$



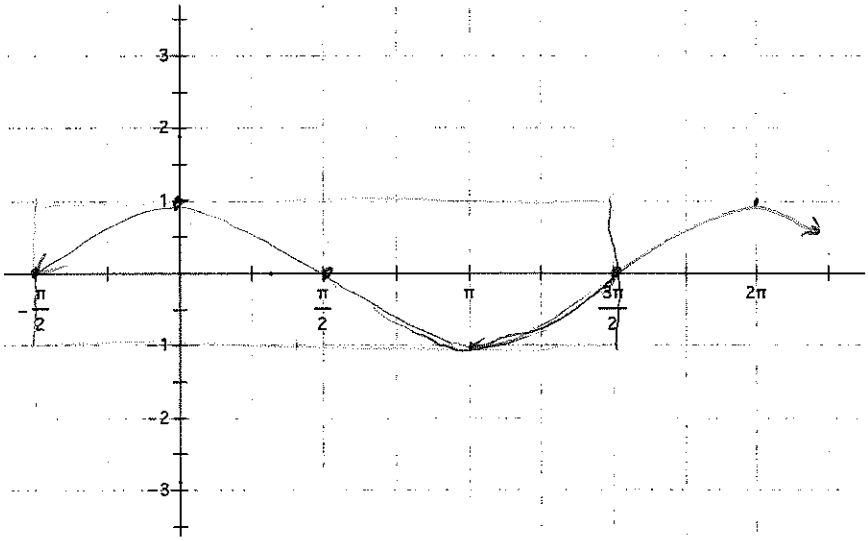
b) $y = \sin(x - \pi)$

right π



c) $y = \sin\left(x + \frac{\pi}{2}\right)$

left $\frac{\pi}{2}$



Homework
Day 1 p.328 #3-54 multiples of three
Day 2 p. 328 #4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52